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Since $x=h$ when $t=0$, $C=\frac{4K\sqrt{h}}{ab\sqrt{2g}}$.

$$\therefore t=T=\frac{4K}{ab\sqrt{2g}}(\sqrt{h}-\sqrt{x}). \quad \therefore x=\left(\frac{4K\sqrt{h}-Tab\sqrt{2g}}{4K}\right)^2.$$

AVERAGE AND PROBABILITY.

98. Proposed by REV. PREBENDARY WHITWORTH, A. M.

A has £ m and B has £ n . They play for points until one of them has lost all his money. If α and β be the respective chances that A and B win any point, the expectation of the number of points played will be

$$\frac{n\alpha^n(\alpha^m-\beta^m)-m\beta^m(\alpha^n+\beta^n)}{(\alpha-\beta)(\alpha^{m+n}-\beta^{m+n})}.$$

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let A_m =A's chance of winning, B_n =B's chance of winning.

Then nA_m, mB_n =A's and B's expectation, respectively.

$\therefore$ Expectation of number of points played= E .

Then $E=(nA_m-mB_n)/(\alpha-\beta)$.

Let A_x =A's chance when he has x pounds and B has $m+n-x$ pounds.

$$\therefore A_x=\frac{\beta}{\alpha+\beta}A_{x-1}+\frac{\alpha}{\alpha+\beta}A_{x+1}.$$

$A_x-A_{x-1}=(\alpha/\beta)(A_{x+1}-A_x)$. Giving x successive values from 1 to x we get $A_1-A_0=(\alpha/\beta)(A_2-A_1)$, $A_2-A_1=(\alpha/\beta)(A_3-A_2)$, etc.

By continued multiplication we get $A_1-A_0=(\alpha/\beta)^{x-1}(A_x-A_{x-1})$ or $A_x-A_{x-1}=(\beta/\alpha)^{x-1}(A_1-A_0)$.

Give x successive values from 1 to x and add

$$A_x-A_0=(A_1-A_0)[1+\beta/\alpha+(\beta/\alpha)^2+\dots+(\beta/\alpha)^{x-1}].$$

But $A_0=0$. $\therefore A_x=A_1[1-(\beta/\alpha)^x]/[1-(\beta/\alpha)]$.

$$A_{m+n}=1. \quad \therefore 1=A_1[\alpha^{m+n}-\beta^{m+n}]/[\alpha^{m+n-1}(\alpha-\beta)].$$

$$\therefore A_1=[\alpha^{m+n-1}(\alpha-\beta)]/(\alpha^{m+n}-\beta^{m+n}).$$

$$\therefore A_x=[\alpha^{m+n-1}(\alpha^x-\beta^x)]/[\alpha^{x-1}(\alpha^{m+n}-\beta^{m+n})].$$

$$\therefore A_m=[\alpha^n(\alpha^m-\beta^m)]/(\alpha^{m+n}-\beta^{m+n}).$$

$$\text{Similarly, } B_n=[\beta^m(\alpha^n-\beta^n)]/(\alpha^{m+n}-\beta^{m+n}).$$

$$\therefore E=\frac{n\alpha^n(\alpha^m-\beta^m)-m\beta^m(\alpha^n-\beta^n)}{(\alpha-\beta)(\alpha^{m+n}-\beta^{m+n})}.$$

99. Proposed by E. B. SEITZ.

A point is taken at random in the surface of a given circle, and from it a line equal in length to the radius is drawn, so as to lie wholly in the surface of the circle. Find the chance that the line intersects in a given diameter. [No. 135, *The Analyst*.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let P be the random point, and AB the given diameter.

$CP=x$, $\angle PCB=\theta$. With P as center and radius= r , the radius of the given circle. Draw the arc DEF .

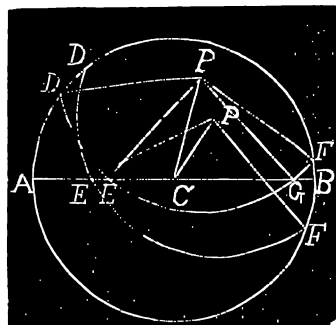
From $\theta=0$ to $\theta=\cos^{-1}\left(\frac{x}{2r}\right)$, F is below AB .

From $\theta=\cos^{-1}\left(\frac{x}{2r}\right)$ to $\theta=\frac{1}{2}\pi$, F is above AB .

Let $\cos^{-1}\left(\frac{x}{2r}\right)=\theta'$, p =the required chance. Then

$$p = \frac{\int_0^r \int_0^{\theta'} EF \cdot x dx d\theta + \int_0^r \int_{\theta'}^{\frac{1}{2}\pi} EG \cdot x dx d\theta}{\int_0^r \int_0^{\frac{1}{2}\pi} DF \cdot x dx d\theta}.$$

$$\begin{aligned} \text{Arc } EF &= r \left[\theta - \sin^{-1} \left(\frac{x}{r} \sin \theta \right) + \cos^{-1} \left(\frac{x}{2r} \right) \right], \text{ arc } DF \\ &= 2r \cos^{-1} \left(\frac{x}{2r} \right), \text{ arc } EG = r \left[\pi - 2 \sin^{-1} \left(\frac{x}{r} \sin \theta \right) \right]. \end{aligned}$$



$$\therefore p = \frac{\int_0^r \left(\int_0^{\theta'} EF d\theta + \int_{\theta'}^{\frac{1}{2}\pi} EG d\theta \right) x dx}{\int_0^r \int_0^{\frac{1}{2}\pi} 2r \cos^{-1} \left(\frac{x}{2r} \right) x dx d\theta}$$

$$\begin{aligned} &= \frac{12}{\pi r^2 (4\pi - 3\sqrt{3})} \int_0^r \left(\int_0^{\theta'} \left[\theta - \sin^{-1} \left(\frac{x}{r} \sin \theta \right) + \cos^{-1} \left(\frac{x}{2r} \right) \right] d\theta \right. \\ &+ \left. \int_{\theta'}^{\frac{1}{2}\pi} \left[\pi - 2 \sin^{-1} \left(\frac{x}{r} \sin \theta \right) \right] d\theta \right) x dx = \frac{12}{\pi r^2 (4\pi - 3\sqrt{3})} \left[\int_0^r \int_0^{\theta'} \left[\theta - \sin^{-1} \left(\frac{x}{r} \sin \theta \right) \right. \right. \\ &+ \left. \left. \cos^{-1} \left(\frac{x}{2r} \right) \right] x dx d\theta + \int_0^r \int_{\theta'}^{\frac{1}{2}\pi} \left[\pi - 2 \sin^{-1} \left(\frac{x}{r} \sin \theta \right) \right] x dx d\theta \right. \\ &\quad \left. - \int_0^r \int_0^{\theta'} \left[\pi - 2 \sin^{-1} \left(\frac{x}{r} \sin \theta \right) \right] x dx d\theta \right] \\ &= \frac{12}{\pi r^2 (4\pi - 3\sqrt{3})} \left[\int_0^r \int_0^{\theta'} \left[\theta - \pi + \cos^{-1} \left(\frac{x}{2r} \right) \right] x dx d\theta + \pi \int_0^r \int_0^{\frac{1}{2}\pi} x dx d\theta \right. \\ &\quad \left. + \int_0^{\frac{1}{2}\pi} \int_0^{2r \cos \theta} \sin^{-1} \left(\frac{x}{r} \sin \theta \right) d\theta x dx - 2 \int_0^{\frac{1}{2}\pi} \int_0^r \sin^{-1} \left(\frac{x}{r} \sin \theta \right) d\theta x dx \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{12}{\pi r^2 (4\pi - 3\sqrt{3})} \left[\int_0^r \left(\frac{3}{2} (\cos^{-1} \frac{x}{2r})^2 - \pi \cos^{-1} \frac{x}{2r} + \frac{1}{2} \pi^2 \right) x dx \right. \\
&\quad \left. + \int_0^{\frac{3}{2}\pi} (4r^2 \theta \cos^2 \theta + \frac{1}{2} r^2 \cot \theta \cos 2\theta - \frac{1}{2} r^2 \theta \operatorname{cosec}^2 \theta) d\theta \right. \\
&\quad \left. - 2 \int_0^{\frac{1}{2}\pi} (\frac{1}{2} r^2 \theta + \frac{1}{2} r^2 \cot \theta - \frac{1}{2} r^2 \theta \operatorname{cosec}^2 \theta) d\theta \right]. \\
&\therefore p = \frac{7\pi^2 + 8\pi\sqrt{3} - 54}{3\pi(4\pi - 3\sqrt{3})}.
\end{aligned}$$

NOTE.—This is problem 135 of *The Analyst*. Quite a discussion arose at the time of its first appearance, in which such eminent mathematicians of Prof. Benj. Pierce, of Harvard, took part. An incomplete solution by Professor Heaton was published. We believe Professor Zerr's solution is correct. ED. F.

100. Proposed by LON C. WALKER, Assistant in Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.

Required the average distance between two points in opposite sides of a regular $2n$ -gon.

Solution by J. SCHEFFER, A. M., Hagerstown, Md., and the PROPOSER.

From any point P at a distance x from the vertex A , in the side $AB=a$ of the regular polygon, draw PE perpendicular to the opposite side DC .

Let Q be any point in DC between D and E at the distance $=y$ from E . Then we have

$$PQ = \sqrt{a^2 \cot^2 \frac{\pi}{2n} + y^2}.$$

The average distance of all points in DE from the point P will be the same as the required average when the line PE is made to move parallel to itself the entire length of a side of the polygon.

The elements of AB and CD at the points P and Q are dx and dy , respectively. Therefore the required mean is

$$\begin{aligned}
M &= \frac{\int_0^a \int_0^x \sqrt{a^2 \cot^2 \frac{\pi}{2n} + y^2} \, dx dy}{\int_0^a \int_0^x dx dy} \\
&= \frac{2}{a^2} \int_0^a \int_0^x \sqrt{a^2 \cot^2 \frac{\pi}{2n} + y^2} \, dx dy = \frac{1}{a^2} \int_0^a \left[x \sqrt{a^2 \cot^2 \frac{\pi}{2n} + x^2} \right. \\
&\quad \left. + a^2 \cot^2 \frac{\pi}{2n} \log \left(\frac{x + \sqrt{a^2 \cot^2 \frac{\pi}{2n} + x^2}}{a \cot \frac{\pi}{2n}} \right) \right] dx = \left[\frac{1}{2} \operatorname{cosec} \frac{\pi}{2n} - \frac{3}{2} \tan \frac{\pi}{4n} \cot^2 \frac{\pi}{2n} \right]
\end{aligned}$$